

The cuprate superconductors: a phenomenological overlook

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Abstract

We, basing on the quantum critical point (QCP)($p = 0.19$), propose a phenomenological description on high- T_c superconductivity in the cuprate superconductors, and suggest it divides the whole doping region into two parts: the underdoped region ($p < 0.19$) and the overdoped region ($p > 0.19$). The electrons in the former are localized and form the localized Fermi liquid, a kind of the non-Fermi liquid, where the carriers are the holes; the electrons in the latter are itinerant and form the Fermi liquid, where the carriers are the electrons.

We further argue that localization is a prerequisite to the pseudogap; the superconductivity gap and coherence forms at the same time, which both share the same energy scale; coherence induces phonon around the node.

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In 1986 J.G. Bednorz and K.A. Müller discovered highcritical temperature (T_c) superconductivity[1], which opened a new chapter in strongly correlated system. They found an onset of the superconductivity in $\text{La}_{2-x}\text{Ba}_x\text{CuO}_4$ near 30°K, modest but still far above the previous values observed in the conventional metals. In the following months, it continued to ascend: to 45°K, to 52°K, and so far as to reach 93°K in $\text{YBa}_2\text{Cu}_3\text{O}_{7-y}$ [2]. But astonishment just began to unveil. In the following years, experimentlists found a series of fancy features, such as pseudogap, stripe and so on.

The great progress in experiment not only provides a wide stage for theorists, but also places a heavy burden on their shoulders. With the push from experiment, the theorists have done a great many of works and reached a lot of consensuses, but are still facing a great challenge. On the front of the experimental phenomena a bewildering variety of theoretical models fall short of our expectations. A unified interpretation on the phase diagram, for example, is still out of control, and even the phase diagram itself is still in dispute.

What is the cornerstone to support so many strange properties in the cuprate superconductors? It is difficult, due to the complexity, to cover all the experimental phenomena at one blow. We have to curtail the branches and concentrate ourselves in some extreme doping cases, for doping dominates the properties of the cuprate superconductors.

I. DOPING

Fundamentally different from a conventional (band) insulator, the parent compounds of the cuprate superconductors is a Mott insulator[3]. Even it is half-filled, one electron per unit cell and the other on-site orbit idle, it cannot conduct. To a conventional insulator, however, such idle orbit can be occupied and used to conduct electricity. It is the strong on-site electron-electron repulsion that blocks such motion and makes the electrons *localized*. Does such localization still exist in the doping compounds?

We consider two ultra doping cases: the ultra-low doping case ($p \sim 2\%$) and the ultra-high doping case.

With the introduction of very few holes, most of the electrons are still localized and only some electrons around holes can move by hopping to form conductivity: electrons form a *localized Fermi liquid*, a kind of the non-Fermi liquid. The carriers thus are holes

and their concentration is proportional to the hole concentration p .

In the ultra-high doping case, holes are enough to make electrons live apart, moving with very low mutual interference: electrons are no longer localized and begin to itinerate, their density of possibility filling respectively the whole copper oxide plane. That is to say, the behaviour of electrons is like the Fermi liquid. The carriers here are not holes but electrons and their concentration is proportional to the concentration of electrons $(1 - p)$, not the concentration of holes p .

The above discussion implies the properties of the electrons in two ultra cases are fundamentally different, for one is the non-Fermi liquid and the other is the Fermi liquid. What is responsible for the difference? Doping! Although we just consider two ultra cases, it is reasonable to speculate that the doping will lead the cuprate superconductors to evolve from the non-Fermi liquid to the Fermi liquid in the whole doping region. Therefore there is a point, at least, to distinguish the two kinds of liquid, on both sides of which, the physical properties should be fundamentally different. But where is it? For now we cannot solve analytically the Hubbard model in the strongly correlated system, we do not know the exact position of this point and have to refer ourselves to experiment.

In fact, Tallon and Loram *et al.* in 1999 had been already aware of the existence of such point and called it *quantum critical point* (QCP)[4]. Subsequently they surveyed all the experimental results, which confirm its existence: the physical properties on both sides of it are fundamentally different[5]. Their works give us an explicit answer: the critical point is in $p = 0.19$ and it is $p = 0.19$, not the optimal doping point $p = 0.16$, that divides the whole doping region into two parts: the underdoped region with $p < 0.19$ and the overdoped region with $p > 0.19$.

The experiment by Uchida[6] also confirms that the carrier concentration in the underdoped region is proportional to the hole concentration (p) and that in the overdoped region is proportional to the electron concentration $(1 - p)$.

In a summary, the electrons in the underdoped region perform as the non-Fermi liquid and the electrons in the overdoped region perform as the Fermi liquid. The former are localized by the on-site Coulomb repulsion and pile up one by one, more similar to the solid; the latter are not localized even with the on-site Coulomb repulsion, more similar to the gas. We will see it is the fundamental starting point to discuss the exotic properties in the cuprate superconductors. In the following we will discuss the doping effects on gap.

II. PSEUDOGAP AND SUPERCONDUCTIVITY GAP

A. Pseudogap

In the conventional superconductors, interaction by exchanging phonon induces electrons to pair as long as temperature drops low enough, which forms quasiparticles and decreases the energy of the system[7]. The expectation of the paired operators in the superconductivity state does not vanish and can be defined as the *energy gap*, an energy scale to determine the critical temperature (T_c)[8]. In the common views, the quasiparticles arising from the paired electrons, furthermore, are in the boson mode and can form coherence at low enough temperature, which implies there is a new energy scale independent of pairing. It is considered that the pairing energy scale is lower than the coherence energy scale in the conventional superconductors and pairing once forms, quasiparticles instantly form coherence and superconductivity emerges; but in the cuprate superconductors, the pairing energy scale, on the contrary, is higher than the coherence energy scale, as a result, the gap in the normal state, known as the *pseudogap*, a precursor to the superconductivity gap, forms before coherence. All reviewed here are called the *precursor pairing scenario* in the literature[9].

It is not difficult, in fact, to check such scenario by experiment, or more explicitly, to see what happens when coherence is destroyed by strong external field. Strong magnetic fields experiment[10] tells us the pseudogap still exists below the critical temperature (T_c) in the underdoped region, when even the superconductivity gap, or the coherence, is violated; there is no any gap, moreover, observed in the overdoped region in the same condition, totally different from the precursor pairing scenario.

What is the physical reason for the pseudogap? We will see in the following that localization is responsible for the pseudogap.

The electrons in the underdoped region, due to the on-site Coulomb repulsion, are localized, or partly localized at least, as discussed above. To decrease the energy of the system with superexchange interaction, two adjacent electrons overlap by resonating to form the singlet, very similar with Anderson's RVB scenario[3]. But it is only the adjacent completely-localized electrons, the electrons that cannot hop, not all the electrons, that form the singlet, which is a little different from the RVB scenario or the precursor pairing

scenario.

Pairing makes the paired electrons inseparable and excludes the holes, which only hop around them. That is to say, the paired electrons are *frozen*. Localization sustains the stability of pairing, and in return, pairing strengthens localization. Were electrons itinerant or movable, they would move apart and be free of the previous superexchange interaction, which is fatal to the adjacent pairing.

But not all the electrons are localized. A hole not only provides the electrons around it opportunities to hop, but is also destructive to their localization, and, furthermore, to their pairing. It manifests the carriers do not pair and the pseudogap pairing is not the precursor to the superconductivity pairing.

In a summary, the pseudogap, or the normal-state gap, is only related to the localized Fermi liquid in the underdoped region and is not found in the Fermi liquid in the overdoped region, as expected. In the copper oxide planes all the paired electrons in the pseudogap state form an *electron glacier* with the help of localization, in which the unpaired electrons flow across the holes. Doping increases the holes, which is destructive to localization, and more to the electron ice. When the doping concentration $p = 0.19$, the electron glacier avalanches and the electrons' flow is free of the constraint from the holes. The electrons begin to perform as the Fermi liquid.

In the Fermi liquid such mechanism cannot apply, for the electrons are no longer localized but itinerant in the lattice and cannot pair respectively via the superexchange interaction, or even they pair, they can easily fall apart. It can be understood by considering only two electrons in the lattice: they, due to the principle of uncertainty, exist in any possible lattice and obtain very low possibility to pair in the neighborhood. To sustain such pairing, a collective mechanism is needed, which is *coherence*.

B. Coherence and superconductivity gap

Coherence originates from the identity of the quasiparticles, which consist of even fermions and are in boson mode. The coherence-induced pairing, instead of existing in partial electrons, applies to all electrons. The electrons in coherence no longer move solely but participate the collective resonance mode as quasiparticles. The emergence of coherence implies electrons are in superconductivity.

Coherence in superconductivity shares the striking similarity with that in the Bose-Einstein condensation (BEC), but also distinguishes itself from the latter, which supports all the peculiarities shown in the cuprate superconductors and also misleads the physicist.

We discuss the distinction between them first.

The chemical potential of the quasiparticles vanishes because the number of the excited quasiparticles is not fixed. There is no constraint, therefore, from the density of particles, which gives the relation between the critical temperature and the particle density in BEC. As a result, *all the energy scale is included in the superconductivity gap and there is no independent energy scale corresponding to coherence.* Though pairing of the carriers and coherence are considered as two main characteristics of superconductivity, it is that one implies two and two merges into one:

The pairing of carriers implies coherence, and vice versa.

Like the role of localization for the pseudogap in the underdoped region, it is coherence that makes paired carriers move together in the superconductivity state and sustains the stability of pairing, whether in the underdoped region or in the overdoped region. The pairing of carriers provides the prerequisite to coherence and in return, coherence keeps pairing stable; they are interdependent and indispensable. Once coherence is violated, the pairing of the carriers, whether in the underdoped region or in the overdoped region, disappears, so does superconductivity and the superconductivity gap[10].

In the overdoped region, the electrons as carriers form pairing and the superconductivity gap, the only candidate for the gap. But in the underdoped region, it becomes a little complicated. The paired carriers in the superconductivity state cover both the pairs with pseudogap and the pairs which are single in the normal state, which coexist and do not compete with other. Two kinds of quasiparticles participate coherence together, which manifests the quasiparticles are identical in the pairing pattern, as a result of which, the two gaps ought to choose the same wave function to pair and share the same physical properties, as suggested by the experiments[11–13].

C. Coherence-induced phonon

Although the superconductivity in the overdoped region originates from the Fermi liquid, the same background as that in the conventional metals, it identifies itself with new

features, as a result of fitting itself the special condition in the cuprate superconductors.

The quasiparticles, or the paired electrons, even in the Fermi liquid, align in the different lattices and transfer the variation of phase, or the thermal excitation in a new way. Suppose an excitation off the node arises from some quasiparticle, it transfers to another quasiparticle by the identity of boson, and so on, which, sometimes called the *phase fluctuation* in the literature[9], gives an interpretation of collective.

The phase fluctuation, in nature, is a new kind of quasiparticle in superconductivity, very similar with the *phonon* in BEC, or in superfluidity[14, 15], The coherence-induced phonon, responsible for the phonon mode shown in the ARPES experiments[16], dominates the thermal excitation around the node, where the energy of the paired quasiparticle vanishes, though there is no explicit electron-phonon interaction in the cuprate superconductors. The dispersion relation of the phonon in superconductivity is just that of the paired quasiparticle, for the energy of the paired quasiparticles in the node is exactly zero. The anisotropy of the gap implies the anisotropy of phonon.

Consequently, we have to revise the traditional BCS theory to apply in the cuprate superconductors, mainly replacing the fermion excitation of quasiparticles with the excitation of phonon, a kind of boson.

Numerical result in rough approximation demonstrates the universal relation between the critical temperature (T_c) and the superconductivity gap (Δ_0), $\frac{2\Delta_0}{k_B T_c} = 4.3$, does not exist, which is substituted by more complicated relationship.

D. Amplitude of the gap

The pairing, whether in the normal state or in the superconductivity state, originates from the resonating between the two adjacent electrons. The amplitude of pairing characterizes the degree of resonating, or more intuitively, the degree of overlapping.

In the pseudogap state, the less the holes, the more crowded the electrons, the more possibility to overlap and the larger the pseudogap; in the superconductivity state, the less the carriers, the more fragile coherence, the less possibility to overlap and the smaller superconductivity gap.

The crowdedness in the underdoped region does not change with temperature, therefore, nor does the amplitude of the pseudogap; but in the overdoped region, it becomes

a little different: the thermal excitation leads to the excitation of phonon, which does not affect pairing, therefore, the amplitude of the superconductivity gap, likewise, does not change with temperature. All in all, the amplitude of gap, no matter which kind of gap, does not change with temperature, as suggested by the experiment[12].

Compared with the pseudogap in the normal state, the superconductivity gap, a product of the collective condensation, is more fragile and therefore, has smaller gap amplitude and lower critical temperature.

III. THE NORMAL STATE

A. Normal state in the overdoped region

In the overdoped region the electrons are in the state of the antiferromagnetic Fermi liquid(AFL), in which the effect of the spin density fluctuations dominates the properties in the thermal equilibrium state and is especially responsible for the linear resistivity[17].

In transportation, electrons avoid the spin-flip process to decrease the resistivity by exchanging the momentum only, leaving the spin unchanged in the scattering, or keeping the original spin order, which, we name as *superexchange transportation*.

B. Normal state in the underdoped region

Though the spin density fluctuations still take effects in the underdoped region, the localization of the electrons plays a decisive role and generates new features, one of which, the pseudogap, for example, has been discussed above, dividing the normal state into two parts:

1. Normal state without the pseudogap

The electrons in the thermal equilibrium without external field can be in spin-disorder, but in transportation the hopping of an isolated hole leaves a line of misaligned spins in its wake[18], which, known as *frustration*, produces a great many of spin-flip processes and consumes additional unnecessary energy. Frustration, therefore, needs to be avoided as possible, thus the charge order and spin order is the only alternative. In some special

doping point, for example, $p = \frac{1}{8}$, frustration vanishes and electrons flow orderly between the domain walls, which is the so-called *stripe*, as discovered in Ref[19]. In the very low doping area, holes are so rare that it is hard to form spin order, as a result, frustration dominates transportation.

To obtain the charge and spin order, massive spin-flip processes are inevitable at the very beginning of transportation, which leads to that the initial resistivity is by far larger than the static resistivity.

Had the charge and spin order been determined, the spin density fluctuations would dominate the properties of the electrons. This is why the resistivity in the normal state without gap shares the similar dependence on temperature, whether in the underdoped region or in the overdoped region.

2. Normal state with the pseudogap

The paired electrons in the normal state, unlike those in the superconductivity state, do not form a collective behaviour. The excitation from the paired quasiparticles melts them into liquid, or depair them. As temperature increases, the quasiparticles vanish gradually. Though the evolution of the pseudogap is different from the superconductivity gap, the critical temperature (T^*) can be calculated by similar method, for the melted electrons are equivalent to the mixture of the quasiparticles in the ground state and in the excited state.

In a summary, the pseudogap is like a floating ice on the Fermi sea in the momentum space and melts gradually till the critical temperature (T^*).

When the electrons form pseudogap, they are frozen and do not participate transportation, they also decouple from other electrons. The spin density fluctuation, therefore, only emerges from the unpaired electrons. As temperature increases, the number of unpaired electrons increases, which makes the resistivity deviate from the linear behaviour[20].

IV. SUMMARY

After above discussion, we summarize our arguments in the following:

1. The hole concentration $p = 0.19$ is a quantum critical point, dividing the whole doping region into two parts: the underdoped region ($p < 0.19$) and the overdoped

region ($p > 0.19$). The electrons in the former are localized and form the localized Fermi liquid, a kind of the non-Fermi liquid, where the carriers are holes; the electrons in the latter are itinerant and form the Fermi liquid, where the carriers are the electrons themselves.

2. Gap is a way to decrease the energy of the system. But different from the pairing in the conventional superconductors, the pairing in the cuprate superconductors only arises from two adjacent electrons and decouple them from the others.

The normal state gap, or the pseudogap, a gap without coherence, only originates from the localized Fermi liquid for localization is a prerequisite to keeping no-coherence pairing stable. The paired quasiparticles in the normal state, instead of vanishing suddenly in the critical point, decrease gradually and vanish finally in the critical temperature.

3. Superconductivity is the condensation of the adjacently-paired carriers, whether in the underdoped region or in the overdoped region. The pairing of the carriers immediately implies coherence. If coherence is violated, the pairing of carriers vanishes.

Coherence induces a new quasiparticle *phonon*, a kind of boson, which is responsible for the excitation of the quasiparticles around the node. The boson distribution, instead of the fermion distribution, therefore, is applied in the calculation.

4. The pseudogap coexists with the superconductivity gap in the superconductivity state of the underdoped region. Once coherence is violated, only the pseudogap is left.
5. A new kind of spin order, stripes, forms in transportation in the underdoped region to avoid frustration.

There are still a lot of issues without definite answers:

1. The properties in the normal state need quantitative results to fit the experimental results, especially in the underdoped region.
2. We believe the pseudogap in other compound, for example, the manganes[21], should share the same physical essence as in the cuprate superconductors, but details need further examination.

3. When stripe forms, the 2-dimension superconductivity turns into 1-dimension superconductivity. Its details also need further examination.

We once expect our suggestion can cover all the issues in the cuprate superconductors and construct an overwhelming theory. But its complexity is beyond our own ability. Even so, we believe it will provide a promising understanding on HTSC. No river, after all, can support a large ship in its origin.

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- [1] J.G. Bednorz and K.A. Müller, Z. Phys. B **64**, 189 (1986);
 - [2] M.K. Wu, J.R. Ashburn, C.J. Torng, P.H. Hor, R.L. Meng, L. Gao, Z.J. Huang, Y.Q. Wang, and C.W. Chu, Phys. Rev. Lett. **58**, 908 (1987).
 - [3] P.W. Anderson, Science **235**, 1196(1987).
 - [4] J.L. Tallon, J.W. Loram, G.V.M. Williams, J.R. Cooper, I.R. Fisher, J.D. Johnson, M.P. Staines, and C. Bernhard, phys. stat. sol. (b) **215**, 531 (1999).
 - [5] J.L. Tallon and J.W. Loram, Physica C **349**, 53(2001).
 - [6] S. Uchida, Physica C **282**, 12(1997).
 - [7] L.N. Cooper, Phys. Rev. **104**, 1189(1956).
 - [8] J.Bardeen, L.N. Cooper, and J.R. Schrieffer, Phys. Rev. **108**, 1175(1957).
 - [9] V.J. Emery and S.A. Kivelson, Nature (London) **374**, 434(1995).
 - [10] Guo-qing Zheng, P.L. Kuhns, A.P. Reyes, B. Liang, and C.T. Lin, Phys. Rev. Lett. **94**, 047006(2005).
 - [11] C.C. Tsuei and J.R. Kirtley, Rev. Mod. Phys. **72**, 969(2000).
 - [12] . Fischer, M. Kugler, I. Maggio-Aprile, C. Berthod and Ch. Renner, Rev. Mod. Phys. **79**, 353(2007) and references therein.
 - [13] K.K. Gomes *et al.* Nature **447**, 569(2007).

- [14] L.D. Landau, Phys. USSR **5**, 71(1941).
- [15] A.J. Leggett, Phys. Rev. Lett. **29**, 1227(1972).
- [16] A. Damascelli, Z. Hussain, and Z.X. Shen, Rev. Mod. Phys. **75**, 473(2003).
- [17] A.J. Millis, H. Monien and D. Pines, Phys. Rev.B **42**, 167(1990).
- [18] J. Orenstein and A.J. Millis, Science **288**, 468(2000).
- [19] J.M. Tranquada, B.J. Sternlieb, J.D. Axe, Y. Nakamura and S. Uchida, Nature **375**, 561(1995).
- [20] T. Timusk and B. Statt, Rep. Prog. Phys. **62**, 61(1999), and references therein.
- [21] D.S. Dessau *et al.* Phys. Rev. Lett. **81**, 192(1998).